Engineering Notes

Three-Fourths T_{max} Rule for Minimum-Area Space Radiators

J. FRANK CONEYBEAR* AND HENRY R. KROEGER†
ASTRA, Inc., Raleigh, N. C.

DETAILED thermodynamic calculations of space power systems can be very laborious. A number of short cuts have been devised, but such computational aids must be used with caution, especially when comparing systems involving different cycle characteristics and/or different working fluids. For example, Moffitt and Klag[1] have demonstrated that if one assumes a Carnot cycle or a saturated Rankine cycle (Fig. 1a) and a radiator that is an abstraction of real radiators (i.e., a radiator without tubes), then the minimum area radiator temperature will be three-fourths of the maximum cycle temperature (in degrees absolute). This $\frac{3}{4}\,T_{max}$ relationship is often used as a rule of thumb. However, it is apparent from Fig. 1a that the saturated Rankine cycle shown is not grossly different from a Carnot cycle, so that the $\frac{3}{4}\,T_{max}$ rule should be applicable. This cycle is quite popular, and working fluids suggested for it include mercury,[2] potassium,[3] rubidium,[4] and, in an unusual version (which expands from saturated to superheated), biphenyl.[5] Even with such cycles, however, efficiency effects tend to change the value for minimum-area radiator temperature, as has been pointed out by Szego.[6]

Finally, there are systems in which the cycle departs so markedly from the Carnot form that it is not even approximately correct to assume that the $\frac{3}{4}\,T_{max}$ relationship gives minimum radiator areas. For example, water, which has been recently proposed,[7] requires a significant amount of superheat in order to achieve competitive weight powerplants. The temperature-entropy diagrams of such cycles can differ markedly in morphology from that of the Carnot cycle, as suggested by Fig. 1b. For such systems, the $\frac{3}{4}\,T_{max}$ relationship is definitely not valid.

As can easily be seen by examination, the assumption that the useful work is proportional to the differences between T_{max} and T_{rad}, as must be assumed to derive the three-fourths rule, must be quite erroneous. To demonstrate this, assume isentropic expansion, as well as a fluid with a very steep slope for the constant pressure line in the superheat region (i.e., very low specific heat). For such a cycle, the area under the superheated "spike" will approach zero, and the temperature that could then be used in the three-fourths assumption with some degree of accuracy would be that of the saturated vapor T_{sat} rather than the top temperature T_{max}.

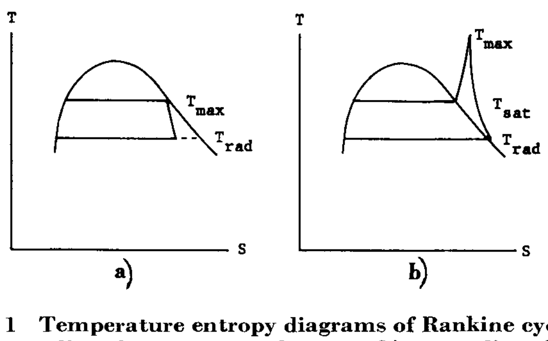

Fig. 1 Temperature entropy diagrams of Rankine cycles: a) expanding from saturated vapor, b) expanding from superheated vapor.

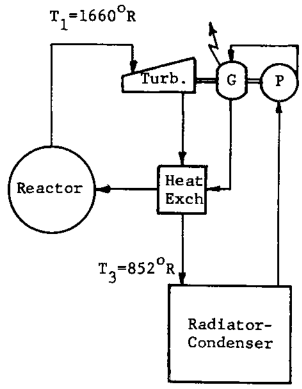

Fig. 2 60 ekw single-loop steam system.[7] Over-all efficiency, 14%; specific weight, 52 lb/ekw; radiator specific weight, 18 lb/ekw; no auxiliary cooling loops.

It is interesting to compare results of these two assumptions with a case (Fig. 2) which has been analyzed in detail.[7] (This 60-ekw powerplant for a synchronous orbit satellite uses steam, with a top temperature of 1660°R, a saturation temperature of 1027°R, and a regenerative heat exchanger between turbine exhaust and reactor inlet.) The results are tabulated in Table 1. Note that the correct temperature for a minimum area radiator is 852°R, which is only 51% of the top temperature.

Table 1 Comparison of values and ratios for three approaches for obtaining the minimum area radiator temperature

Approaches	Data		
Relationship used	$\frac{3}{4}\,T_{max}$	$\frac{3}{4}\,T_{sat}$	Actual min area, T_{rad}
T_{rad}, from relationship °R	1245	770	852
Ratio T_{rad}/T_{max}	0.75	0.46	0.51

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* President. Member AIAA.
† Vice-President.

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